Physics Letters B 757 (2016) 237–243

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Supersymmetric composite gauge fields with compensators

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ARTICLE INFO

Article history:
Received 30 January 2016
Received in revised form 21 March 2016
Accepted 23 March 2016
Available online 29 March 2016
Editor: M. Cvetič

Keywords:
Composite gauge fields
$N = 1$ supersymmetry
Compensator fields
Gauged chiral symmetry
Axial-vector multiplet

ABSTRACT

We study supersymmetric composite gauge theory, supplemented with compensator mechanism. As our first example, we give the formulation of $N = 1$ supersymmetric non-Abelian composite gauge theory without the kinetic term of a non-Abelian gauge field. The important ingredient is the Proca–Stueckelberg-type compensator scalar field that makes the gauge-boson field equation non-singular, *i.e.*, the field equation can be solved for the gauge field algebraically as a perturbative expansion. As our second example, we perform the gauging of chiral-symmetry for $N = 1$ supersymmetry in four dimensions by a composite gauge field. These results provide supporting evidence for the consistency of the mechanism that combines the composite gauge field formulations and compensator formulations, all unified under supersymmetry.

1. Introduction

The first work for the supersymmetric formulation of composite gauge field seems to be the paper by B. Milewski [1], where a gauge field is defined as the quadratic form of a scalar field sandwiching a derivative, given schematically by $A_\mu = i(\varphi^\dagger \partial_\mu \varphi)$. Accordingly, the covariant derivative on φ is

$$D_\mu \varphi \equiv \partial_\mu \varphi + i(i\varphi^\dagger \partial_\mu \varphi)\,\varphi \ . \tag{1.1}$$

The noteworthy feature of such a construct is the appearance of a cubic interaction. In practice, this sort of system is cumbersome to handle, because of cubic-interaction terms.

Independent of such supersymmetric composite gauge field theory constructions, the supersymmetrization [2,3] of Proca–Stuckelberg (compensator) formulation [4] has also been achieved. This compensator formulation has been also applied to the gauging of the dilaton-shift symmetry in $N = 1$ supergravity in four dimensions (4D) [5]. These formulations are all inspired by recent developments in supersymmetric tensor-hierarchy formulation of consistent interactions for non-Abelian tensor fields [6,7].

According to the bosonic non-Abelian compensator mechanism [8], the scalar field φ^I with the adjoint index I couples to the non-Abelian gauge field as

$$P_\mu{}^I \equiv \big[\,(\partial_\mu e^{\varphi})e^{-\varphi}\,\big]^I + m A_\mu{}^I \ , \tag{1.2}$$

so that the kinetic term in the lagrangian is $-(1/2)(P_\mu{}^I)^2$, which is at most quadratic in $A_\mu{}^I$. The advantage of this formulation is that the $A_\mu{}^I$-field equation is[1]

$$\begin{aligned} A_\mu{}^I &\doteq -m^{-1}\big[\,(\partial_\mu e^{\varphi})\,e^{-\varphi}\big]^I + J_\mu{}^I \\ &= -m^{-1}\partial_\mu \varphi^I + J_\mu{}^I + \mathcal{O}(\Phi^2) \ , \end{aligned} \tag{1.3}$$

where $J_\mu{}^I$ is other current terms, while $\mathcal{O}(\Phi^2)$ are quadratic terms in fundamental fields. The expansion (1.3) starts with $\partial_\mu \varphi^I$, when the exponential function is expanded. The $J_\mu{}^I$ is typically made of fermionic fields, coming from their kinetic terms. As is easily seen, the lowest-order term in (1.3) directly gives the leading term $\partial_\mu \alpha^I$ of the gauge transformation of $A_\mu{}^I$. Accordingly, all lagrangian terms become quadratic, which are easier to handle. Even though the composite-field formulation in [3] also has some non-polynomial terms, these terms are all related to non-Abelian couplings, in contrast with the conventional formulation such as (1.1), where even the lowest-order term is already quadratic.[2] Another drawback in the conventional composite gauge field formulation is that the gauge field equation results in the vanishing of its source current, that restricts the dynamical freedom of source fields. If we

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[1] We use the symbol $\doteq$ for field equations.

[2] More detailed explanation is found in the next section.

http://dx.doi.org/10.1016/j.physletb.2016.03.062

adopt the compensator formulation, there is the advantage of 'linearizing' the composite field, that avoids higher-order terms such as (1.1), or the introduction of unnecessary constraints on fields.

Based on this philosophy, we formulate in this paper two models of supersymmetric composite gauge field theories in four dimensions (4D), following the compensator mechanisms in [3,5]. Our first model has the field content with the following three multiplets: (i) Vector multiplet (VM) $(A_\mu{}^I, \lambda^I, C_{\mu\nu\rho}{}^I)$, (ii) Compensator tensor multiplet (CTM) $(B_{\mu\nu}{}^I, \chi^I, \varphi^I)$, and (iii) Chiral multiplet (CM) $(A^i, B^i, \psi^i, F^i, G^i)$. Since our objective is to work with *composite*-gauge fields, we do *not* supply the kinetic terms for the VM.

Our second model has the VM $(A_\mu, \lambda, C_{\mu\nu\rho})$ plus CTM $(B_{\mu\nu}, \chi, \varphi)$ coupled to chiral multiplet (CM) (A, B, ψ, F, G) in 4D. The global chiral-symmetry of the CM is gauged with the axial vector A_μ with the compensator pseudo-scalar φ. We show that the compensator formulation for the chiral-symmetry indeed works *without* the kinetic term of the VM, namely, the composite gauge formulation is shown to be possible even for chiral-symmetry of $N = 1$ supersymmetry.

The formulation of 'composite gauge' fields in supersymmetric theories is *not* new. For example, in the context of $N = 8$ supergravity in 4D [9], its scalar fields arrange themselves to form a *composite* gauge field for the isotropy group $SU(8)$ of $E_{7(+7)}/SU(8)$, as a *hidden* gauge symmetry. There are differences as well as similarities, between our formulation and [9]. One similarity is that our gauge fields have *no* kinetic term as in [9]. The difference is that our gauge fields are regarded as an *independent* fields from the outset, with *no* rearrangement by scalars is needed to form the composite gauge fields as *hidden* symmetry. In this sense, our system with *manifest* symmetry is easier to handle than *hidden* symmetries. In a sense, our approach is similar to the so-called 1st-order formalism in supergravity [10].

In the next section, we describe the purely bosonic case of composite gauge field mechanism, based on a compensator formulation. In section 3, by using a compensator, we perform the typical supersymmetric formulation of non-Abelian composite gauge field with global $N = 1$ supersymmetry. In section 4, we present another example of gauging the chiral-symmetry [11] of an $N = 1$ chiral multiplet in 4D, by a composite gauge field combined with a compensator. Section 5 is devoted to the concluding remarks.

2. Scalar compensator for composite gauge field

We first clarify the crucial role played by the compensator field in the composite gauge field formulation. In elucidating the role of a compensator scalar field for a composite gauge field, we employ a toy model with the field content consisting of the non-Abelian gauge field $A_\mu{}^I$, a compensator real scalar field φ^I, another real scalar field ϕ^i and a Majorana field ψ^i. For simplicity, we consider the gauge group $SO(\mathcal{N})$, and the index I (or i) is for the adjoint (or vectorial) representation, so that all generators are antisymmetric: $(T^I)^{jk} = -(T^I)^{kj}$.

Consider our action $I_0 \equiv m^2 \int d^4x\, \mathcal{L}_0$ with the lagrangian[3]

$$\mathcal{L}_0 = -(P_\mu{}^I)^2 - \tfrac{1}{2}(D_\mu\phi^i)^2 + \tfrac{1}{2}(\overline{\psi}^i \not{D}\psi^i) \ , \tag{2.1}$$

where there is *no* kinetic term for the gauge field $A_\mu{}^I$, because the gauge field is taken to be *composite*. The $P_\mu{}^I$ is the field strength of the compensator field φ^I, defined by

$$P_\mu{}^I \equiv \left[(\partial_\mu e^\varphi)\, e^{-\varphi} \right]^I + mA_\mu{}^I \equiv P_\mu^{(0)\,I} + mA_\mu{}^I \ . \tag{2.2}$$

The infinitesimal gauge transformation rule of the non-Abelian group is

$$\begin{aligned}
&\delta_T (e^\varphi)^I = -m(\alpha e^\varphi)^I \ , \\
&\delta_T A_\mu{}^I = D_\mu \alpha^I \equiv \partial_\mu \alpha^I + m f^{IJK} A_\mu{}^J \alpha^K \ , \qquad (2.3a)\\
&\delta_T \phi^i = -m(T^J)^{ik} \alpha^J \phi^k \ , \\
&\delta_T \psi^i = -m(T^J)^{ik} \alpha^J \psi^k \ . \qquad (2.3b)
\end{aligned}$$

It is easy to see that $\delta_T P_\mu{}^I = -m f^{IJK} \alpha^J P_\mu{}^K$.

The field equation of $A_\mu{}^I$ is

$$P_\mu{}^I \doteq +\tfrac{1}{2}(\overline{\psi}\gamma_\mu T^I \psi) + \tfrac{1}{2}(\overline{\phi} T^I D_\mu \phi) \ , \tag{2.4}$$

where $(\overline{\psi}\gamma_\mu T^I\psi) \equiv (T^I)^{jk}(\overline{\psi}^j\gamma_\mu\psi^k)$, and $(\phi T^I D_\mu\phi) \equiv (T^I)^{jk}\phi^j D_\mu\phi^k$.

The importance of φ is now elucidated. Note that there is a linear $A_\mu{}^I$-field involved in $P_\mu{}^I$ on the LHS. This enables us to solve (2.4) *locally and algebraically* for $A_\mu{}^I$:

$$\begin{aligned}
&mM^{IJ} A_\mu{}^J \doteq -P_\mu^{(0)\,I} + \tfrac{1}{2}(\overline{\psi}\gamma_\mu T^I\psi) + (\phi T^I \partial_\mu \phi) \\
&\Longrightarrow \\
&A_\mu{}^I \doteq m^{-1}(M^{-1})^{IJ} \left[-P_\mu^{(0)\,J} + \tfrac{1}{2}(\overline{\psi}\gamma_\mu T^J\psi) + (\overline{\phi} T^J \partial_\mu\phi) \right] \\
&\qquad = m^{-1}(I + N + N^2 + \cdots)^{IJ} \\
&\qquad\quad \times \left[-\left\{ (\partial_\mu e^\varphi) e^{-\varphi} \right\}^J + \tfrac{1}{2}(\overline{\psi}\gamma_\mu T^J\psi) + (\overline{\phi} T^J \partial_\mu\phi) \right] ,
\end{aligned} \tag{2.5}$$

where $P_\mu^{(0)\,I}$ is defined in (2.2), while $M^{IJ} \equiv \delta^{IJ} - (\phi T^I T^J \phi) \equiv \delta^{IJ} - N^{IJ}$. The crucial ingredient here is the existence of the unit matrix I inside $M = I - N$ that enables us to give the expression for $A_\mu{}^I$ explicitly as an infinite but perturbative series in terms of the matrix N.

This is very contrastive to the usual case without the compensator field, because in the latter, the unit-matrix in $M = I - N$ is absent, so that $M^{-1} = -N^{-1}$ becomes *singular* in the zero-field limit $\phi \to 0,\ N \to 0,\ M^{-1} \to \infty$. The drawback of such singularity is the lack of perturbative treatment of the usual formulation. To the contrary in our formulation, the composite gauge field $A_\mu{}^I$ in (2.5) is *non-singular*, so it is compatible with the perturbation around the zero v.e.v. limits.

The 'gauge field' of (2.5) indeed transforms as the covariant gradient:

$$\delta_T A_\mu{}^I \doteq D_\mu \alpha^I \ . \tag{2.6}$$

Here we used the symbol $\doteq$, due to the $A_\mu{}^I$-field equation (2.5) used. This can be confirmed by the following useful lemmas:

$$\begin{aligned}
&\delta_T P_\mu^{(0)I} = -m\partial_\mu\alpha^I - m f^{IJK}\alpha^J P_\mu^{(0)K} \ , \\
&\delta_T N^{IJ} = -2 f^{KL(I|} \alpha^K N^{|J)L} \ , \qquad (2.7a)\\
&\delta_T (N^p)^{IJ} = -2 f^{KL(I|} \alpha^K (N^p)^{|J)L} \qquad (p = 1, 2, 3, \cdots) \ , \qquad (2.7b)\\
&\delta_T (M^{-1})^{IJ} = -2 f^{KL(I|} \alpha^K (M^{-1})^{|J)L} \ . \qquad (2.7c)
\end{aligned}$$

[3] Our lagrangian has the dimension of $(\text{mass})^2$, so that there is the factor m^2 needed in the action in front of the lagrangian. Relevantly, our fundamental fermionic (or bosonic) fields have dimension $m^{1/2}$ (or m^0), so that the minimal gauge-coupling constant has the mass dimension m.

Relevantly, we can confirm that our composite gauge field (2.5) has a *non-vanishing* field strength, that can *not* be gauged away. For simplicity to see this, we truncate ϕ^i, keeping only ψ[4]:

$$A_\mu{}^I \doteq -m^{-1} P_\mu^{(0)I} + \tfrac{1}{2} m^{-1} (\overline{\psi}^i \gamma_\mu T^I \psi) \tag{2.8}$$

leading to[5]

$$\begin{aligned} 0 \stackrel{?}{=} F_{\mu\nu}{}^I &\equiv +2\partial_{[\mu} A_{\nu]}{}^I + m f^{IJK} A_\mu{}^J A_\nu{}^K \\ &\doteq -2m^{-1} (\overline{\psi} \gamma_{[\mu} T^I D_{\nu]} \psi) \\ &\quad + 2m^{-1} f^{IJK} (\overline{\psi} \gamma_{[\mu} T^J \psi) P_{\nu]}^{(0)K} + \mathcal{O}(\psi^4) \neq 0 \ , \end{aligned} \tag{2.9}$$

where we used $(\overline{\psi} \gamma_\mu T^I T^J \psi) = \frac{1}{2} f^{IJK} (\overline{\psi} \gamma_\mu T^K \psi)$.

In the usual case with *no* compensator field φ^I, we rely on a Higgs mechanism, accompanied by the non-zero v.e.v. for ϕ^i. Accordingly, the matrix M in (2.5) gains the v.e.v. term, playing a role similar to our compensator φ^I. However, our formulation with the compensator mechanism is much easier, because we have *neither* to shift v.e.v.s, *nor* to break gauge symmetry.

3. Supersymmetric composite gauge field

In the previous section, we have seen the crucial role played by the compensator φ^I. In this section, we supersymmetrize such a non-supersymmetric system.

As has been alluded to previously, we consider three multiplets: (i) VM $(A_\mu{}^I, \lambda^I, C_{\mu\nu\rho}{}^I)$, (ii) CTM $(B_{\mu\nu}{}^I, \chi^I, \varphi^I)$, and (iii) CM $(A^i, B^i, \psi^i, F^i, G^i)$. The $C_{\mu\nu\rho}{}^I$-field in the VM is kind of auxiliary, dual to the conventional D-auxiliary field. The two multiplets VM and CTM are for the supersymmetric compensator formulation described in our previous paper [12]. Compared with [12], the only difference is the existence of the CM that gives the terms like $(\phi T T \phi)$ or the $(\overline{\psi} T D \psi)$-terms in the $A_\mu{}^I$-field equations, as we saw in section 2.

After all, our total action $I_1 \equiv m^2 \int d^4 x \mathcal{L}_1$ has the lagrangian

$$\begin{aligned} \mathcal{L}_1 = &- \tfrac{1}{12} (G_{\mu\nu\rho}{}^I)^2 + \tfrac{1}{2} (\overline{\chi}^I \not{D} \chi^I) - \tfrac{1}{2} (P_\mu{}^I)^2 + m (\overline{\lambda}^I \chi^I) \\ &- \tfrac{1}{2} (D_\mu A^i)^2 - \tfrac{1}{2} (D_\mu B^i)^2 + \tfrac{1}{2} (\overline{\psi}^i \not{D} \psi^i) \\ &+ \tfrac{1}{2} (F^i)^2 + \tfrac{1}{2} (G^i)^2 + m (T^I)^{jk} (\overline{\lambda}^I \psi^j) A^k \\ &+ i m (T^I)^{jk} (\overline{\lambda}^I \gamma_5 \psi^j) B^k - m (T^I)^{jk} \widetilde{H}^I A^j B^k \ . \end{aligned} \tag{3.1}$$

Since we are aiming for a composite gauge field $A_\mu{}^I$, we do *not* give its kinetic term as per our prescription. This also implies the lack of the kinetic terms for its partner fields λ^I and $C_{\mu\nu\rho}{}^I$. Our field strengths are defined by [12]

$$\begin{aligned} \mathcal{F}_{\mu\nu}{}^I &\equiv +2D_{[\mu} A_{\nu]}{}^I + 2 f^{IJK} A_\mu{}^J A_\nu{}^K + m^{-1} f^{IJK} P_\mu{}^J P_\nu{}^K \\ &\equiv F_{\mu\nu}{}^I + m^{-1} f^{IJK} P_\mu{}^J P_\nu{}^K \ , \end{aligned} \tag{3.2a}$$

$$G_{\mu\nu\rho}{}^I \equiv +3D_{[\mu} B_{\nu\rho]}{}^I + m C_{\mu\nu\rho}{}^I \ , \tag{3.2b}$$

$$H_{\mu\nu\rho\sigma}{}^I \equiv +4D_{[\mu} C_{\nu\rho\sigma]}{}^I + 6 f^{IJK} F_{[\mu\nu}{}^J B_{\rho\sigma]}{}^K \ , \tag{3.2c}$$

$$\widetilde{H}^I \equiv + \tfrac{1}{24} \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho\sigma}{}^I \ , \tag{3.2d}$$

$$P_\mu{}^I \equiv + \left[(D_\mu e^\varphi) e^{-\varphi} \right]^I \equiv \left[(\partial_\mu e^\varphi + A_\mu e^\varphi) e^{-\varphi} \right]^I \ . \tag{3.2e}$$

Even though $\mathcal{F}_{\mu\nu}{}^I$ and $H_{\mu\nu\rho\sigma}{}^I$ are *not* directly involved in the lagrangian $\mathcal{L}_1$, they are important for the invariance $\delta_Q I_1 = 0$, and for the Bianchi identities (BIs)

$$D_{[\mu} \mathcal{F}_{\nu\rho]}{}^I \equiv + f^{IJK} \mathcal{F}_{[\mu\nu}{}^J P_{\rho]}{}^K \ , \tag{3.3a}$$

$$D_{[\mu} G_{\nu\rho\sigma]}{}^I \equiv + \tfrac{1}{4} m H_{\mu\nu\rho\sigma}{}^I \ , \tag{3.3b}$$

$$D_{[\mu} P_{\nu]}{}^I \equiv + \tfrac{1}{2} m \mathcal{F}_{\mu\nu}{}^I \ . \tag{3.3c}$$

Needless to say, there is *no* corresponding BI for $H_{\mu\nu\rho\sigma}{}^I$, because of its highest rank in 4D.

Our action I_1 is invariant under $N = 1$ supersymmetry[6]

$$\delta_Q A_\mu{}^I = +(\overline{\epsilon} \gamma_\mu \lambda^I) - m^{-1} f^{IJK} (\overline{\epsilon} \chi^J) P_\mu{}^K \ , \tag{3.4a}$$

$$\begin{aligned} \delta_Q \lambda^I = &+ \tfrac{1}{2} (\gamma^{\mu\nu} \epsilon) \mathcal{F}_{\mu\nu}{}^I - \tfrac{1}{24} (\gamma^{[4]} \epsilon) H_{[4]}{}^I \\ &- \tfrac{1}{4} f^{IJK} \Big[\epsilon (\overline{\lambda}^J \chi^K) + (\gamma_\mu \epsilon)(\overline{\lambda}^J \gamma^\mu \chi^K) \\ &+ (\gamma_{\mu\nu} \epsilon)(\overline{\lambda}^J \gamma^{\mu\nu} \chi^K) - (\gamma_5 \gamma_\mu \epsilon)(\overline{\lambda}^J \gamma_5 \gamma^\mu \chi^K) \\ &- 3 (\gamma_5 \epsilon)(\overline{\lambda}^J \gamma_5 \chi^K) \Big] \ , \end{aligned} \tag{3.4b}$$

$$\delta_Q C_{\mu\nu\rho}{}^I = +(\overline{\epsilon} \gamma_{\mu\nu\rho} \lambda^I) - 3 f^{IJK} (\delta_Q A_{[\mu}{}^J) B_{\nu\rho]}{}^K \ , \tag{3.4c}$$

$$\delta_Q B_{\mu\nu}{}^I = +(\overline{\epsilon} \gamma_{\mu\nu} \chi^I) \ , \tag{3.4d}$$

$$\delta_Q \chi^I = + \tfrac{1}{6} (\gamma^{[3]} \epsilon) G_{[3]}{}^I - (\gamma^\mu \epsilon) P_\mu{}^I \ , \tag{3.4e}$$

$$\left[(\delta_Q e^\varphi) e^{-\varphi} \right]^I = +(\overline{\epsilon} \chi^I) \ , \tag{3.4f}$$

$$\delta_Q A^i = +(\overline{\epsilon} \psi^i) \ , \qquad \delta_Q B^i = +i (\overline{\epsilon} \gamma_5 \psi^i) \ , \tag{3.4g}$$

$$\begin{aligned} \delta_Q \psi^i = &-(\gamma^\mu \epsilon) D_\mu A^i + i (\gamma_5 \gamma^\mu \epsilon) D_\mu B^i \\ &- \epsilon F^i - i (\gamma_5 \epsilon) G^i \ , \end{aligned} \tag{3.4h}$$

$$\begin{aligned} \delta_Q F^i = &+(\overline{\epsilon} \not{D} \psi^i) + m (T^J)^{ik} (\overline{\epsilon} \lambda^J) A^k \\ &+ i m (T^J)^{ik} (\overline{\epsilon} \gamma_5 \lambda^J) B^k \ , \end{aligned} \tag{3.4i}$$

$$\begin{aligned} \delta_Q G^i = &+i (\overline{\epsilon} \gamma_5 \not{D} \psi^i) + i m (T^J)^{ik} (\overline{\epsilon} \gamma_5 \lambda^J) A^k \\ &- m (T^J)^{ik} (\overline{\epsilon} \lambda^J) B^k \ . \end{aligned} \tag{3.4j}$$

The general transformations for our field strengths are

$$\delta \mathcal{F}_{\mu\nu}{}^I = +2D_{[\mu} (\delta A_{|\nu]}{}^I) + 2m^{-1} f^{IJK} (\delta P_{[\mu}{}^J) P_{\nu]}{}^K \ , \tag{3.5a}$$

$$\begin{aligned} \delta G_{\mu\nu\rho}{}^I &= +3D_{[\mu} (\delta B_{\nu\rho]}{}^I) + \Big[\delta C_{\mu\nu\rho}{}^I + 3 f^{IJK} (\delta A_{[\mu}{}^J) B_{\nu\rho]}{}^K \Big] \\ &\equiv +3D_{[\mu} (\delta B_{\nu\rho]}{}^I) + \widetilde{\delta} C_{\mu\nu\rho}{}^I \ , \end{aligned} \tag{3.5b}$$

$$\begin{aligned} \delta H_{\mu\nu\rho\sigma}{}^I = &+4D_{[\mu} (\widetilde{\delta} C_{\nu\rho\sigma]}{}^I) + 4 f^{IJK} (\delta A_{[\mu}{}^J) G_{\nu\rho\sigma]}{}^K \\ &- 6 f^{IJK} (\delta B_{[\mu\nu}{}^J) F_{\rho\sigma]}{}^K \ , \end{aligned} \tag{3.5c}$$

$$\begin{aligned} \delta P_\mu{}^I = &D_\mu \left[(\delta e^\varphi) e^{-\varphi} \right]^I + f^{IJK} \left[(\delta e^\varphi) e^{-\varphi} \right]^J P_\mu{}^K \\ &+ m \, \delta A_\mu{}^I \ . \end{aligned} \tag{3.5d}$$

As in [12], the three different gauge transformations for the gauge fields $A_\mu{}^I$, $B_{\mu\nu}{}^I$ and $C_{\mu\nu\rho}{}^I$, called δ_T, δ_U and δ_V are needed. Their explicit transformations are

$$\delta_T A_\mu{}^I = +D_\mu \alpha^I \ , \qquad \delta_T (e^\varphi)^I = -m (\alpha e^\varphi)^I \ , \tag{3.6a}$$

$$\begin{aligned} &\delta_T (B_{\mu\nu}{}^I, \ C_{\mu\nu\rho}{}^I, \ \lambda^I, \ \chi^I) \\ &\quad = -f^{IJK} \alpha^J (B_{\mu\nu}{}^K, \ C_{\mu\nu\rho}{}^K, \ \lambda^K, \ \chi^K) \ , \end{aligned} \tag{3.6b}$$

$$\delta_T (A^i, \ B^i, \ \psi^i, \ F^i, \ G^i) = -(T^J)^{ik} \alpha^J (A^k, \ B^k, \ \psi^k, \ F^k, \ G^k) \ , \tag{3.6c}$$

$$\delta_U B_{\mu\nu}{}^I = +2D_{[\mu} \beta_{\nu]}{}^I \ ,$$

$$\delta_U C_{\mu\nu\rho}{}^I = +3 f^{IJK} \beta_{[\mu}{}^J F_{\nu\rho]}{}^K \ , \tag{3.6d}$$

$$\delta_V C_{\mu\nu\rho}{}^I = +3D_{[\mu} \gamma_{\nu\rho]}{}^I \ , \qquad \delta_V B_{\mu\nu}{}^I = -m \gamma_{\mu\nu}{}^I \ . \tag{3.6e}$$

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[4] If we can show that $F_{\mu\nu}{}^I \neq 0$ for the case of $\phi^i = 0$ and $\psi^i \neq 0$, it is enough for the proof of $F_{\mu\nu}{}^I \neq 0$ for $\phi^i \neq 0$ and $\psi^i \neq 0$.

[5] We use a symbol $\stackrel{?}{=}$ for an equality under question.

[6] We use the symbol $[n]$ for totally antisymmetric n indices $\mu_1 \cdots \mu_n$ to save space.

Other fields under δ_U and δ_V, do *not* transform, *e.g.*, $\delta_U A_\mu{}^I = 0$. As a corollary, we get the covariance and invariance of all of our field strengths:

$$\begin{aligned}&\delta_T (P_\mu{}^I, F_{\mu\nu}{}^I, G_{\mu\nu\rho}{}^I, H_{\mu\nu\rho\sigma}{}^I) \\ &\quad = - f^{IJK} \alpha^J (P_\mu{}^K, F_{\mu\nu}{}^K, G_{\mu\nu\rho}{}^K, H_{\mu\nu\rho\sigma}{}^K) \ , \qquad (3.7a)\\ &\delta_U (P_\mu{}^I, F_{\mu\nu}{}^I, G_{\mu\nu\rho}{}^I, H_{\mu\nu\rho\sigma}{}^I) = 0 \ , \\ &\delta_V (P_\mu{}^I, F_{\mu\nu}{}^I, G_{\mu\nu\rho}{}^I, H_{\mu\nu\rho\sigma}{}^I) = 0 \ . \qquad (3.7b)\end{aligned}$$

As the confirmation of the total consistency of our system, we first derive all field equations, and next investigate their consistency. The field equations are

$$\frac{\delta I_1}{\delta \overline{\lambda}^I} = + m \chi^I + m (T^I)^{jk} \psi^j A^k + i m (T^I)^{jk} (\gamma_5 \psi^j) B^k \doteq 0 \ , \qquad (3.8a)$$

$$\frac{\delta I_1}{\delta \overline{\chi}^I} = + \not{D} \chi^I + m \lambda^I \doteq 0 \ , \qquad (3.8b)$$

$$\frac{\delta I_1}{\delta \overline{\psi}^i} = + \not{D} \psi^i + m (T^J)^{ik} \lambda^J A^k + i m (T^J)^{ik} (\gamma_5 \lambda^J) B^k \doteq 0 \ , \qquad (3.8c)$$

$$\begin{aligned}\frac{\delta I_1}{\delta A_\mu{}^I} = & - m P^{\mu\, I} + 3 f^{IJK} B_{\nu\rho}{}^J \left(\frac{\delta I_1}{\delta C_{\mu\nu\rho}{}^K} \right) \\ & - \tfrac{1}{2} m f^{IJK} (\overline{\chi}^J \gamma^\mu \chi^K) + \tfrac{1}{2} m (T^I)^{jk} (\overline{\psi}^j \gamma^\mu \psi^k) \\ & + m (T^I)^{jk} (A^j D^\mu A^k + B^j D^\mu B^k) \\ & - \tfrac{1}{6} \epsilon^{\mu\nu\rho\sigma} f^{IJK} (T^J)^{lm} G_{\nu\rho\sigma}{}^K A^l B^m \doteq 0 \ , \qquad (3.8d)\end{aligned}$$

$$\begin{aligned}\frac{\delta I_1}{\delta B_{\mu\nu}{}^I} = & + \tfrac{1}{2} D_\rho G^{\mu\nu\rho\, I} - \tfrac{1}{4} m \epsilon^{\mu\nu\rho\sigma} f^{IJK} (T^J)^{lm} F_{\rho\sigma}{}^K A^l B^m \\ & \doteq 0 \ , \qquad (3.8e)\end{aligned}$$

$$\begin{aligned}\frac{\delta I_1}{\delta C_{\mu\nu\rho}{}^I} = & - \tfrac{1}{6} m \left[G^{\mu\nu\rho\, I} + \epsilon^{\mu\nu\rho\sigma} (T^I)^{jk} D_\sigma (A^j B^k) \right] \\ & \doteq 0 \ , \qquad (3.8f)\end{aligned}$$

$$\frac{\delta I_1}{[\, (\delta e^\varphi) e^{-\varphi}]^I} = + D_\mu P^{\mu\, I} \doteq 0 \ , \qquad (3.8g)$$

$$\frac{\delta I_1}{\delta A^i} = + D_\mu^2 A^i - m (T^J)^{ik} (\overline{\lambda}^J \psi^k) - m (T^J)^{ik} \widetilde{H}^J B^k \doteq 0 \ , \qquad (3.8h)$$

$$\frac{\delta I_1}{\delta B^i} = + D_\mu^2 B^i - i m (T^J)^{ik} (\overline{\lambda}^J \gamma_5 \psi^k) + m (T^J)^{ik} \widetilde{H}^J A^k \doteq 0 \ , \qquad (3.8i)$$

$$\frac{\delta I_1}{\delta F^i} = + F^i \doteq 0 \ , \qquad \frac{\delta I_1}{\delta G^i} = + G^i \doteq 0 \ . \qquad (3.8j)$$

The consistency of the $A_\mu{}^I$-field equation (3.8d) is confirmed by the vanishing of its divergence by the use of other field equations:

$$\begin{aligned} 0 \overset{?}{=} & + D_\mu \left(\frac{\delta I_1}{\delta A_\mu{}^I} \right) \\ = & - m \Bigg[+ f^{IJK} \left\{ \overline{\lambda}^J \left(\frac{\delta I_1}{\delta \overline{\lambda}^K} \right) \right\} \\ & + f^{IJK} \left\{ \overline{\chi}^J \left(\frac{\delta I_1}{\delta \overline{\chi}^K} \right) \right\} - (T^I)^{jk} \left\{ \overline{\psi}^j \left(\frac{\delta I_1}{\delta \overline{\psi}^k} \right) \right\} \\ & + f^{IJK} B_{\mu\nu}{}^J \left(\frac{\delta I_1}{\delta B_{\mu\nu}{}^K} \right) + f^{IJK} C_{\mu\nu\rho}{}^J \left(\frac{\delta I_1}{\delta C_{\mu\nu\rho}{}^K} \right) \\ & + \frac{\delta I_1}{\{ (\delta e^\varphi) e^{-\varphi} \}^I} - (T^I)^{jk} A^j \left(\frac{\delta I_1}{\delta A^k} \right) - (T^I)^{jk} B^j \left(\frac{\delta I_1}{\delta B^k} \right) \Bigg] \\ \doteq & \ 0 \ , \qquad (3.9)\end{aligned}$$

where $\overset{?}{=}$ is an 'equality under investigation'. In (3.9), we have used field equations *only* at the last equality. We can check similar divergences of the $B_{\mu\nu}{}^I$ and $C_{\mu\nu\rho}{}^I$-field equations:

$$0 \overset{?}{=} D_\nu \left(\frac{\delta I_1}{\delta B_{\mu\nu}{}^I} \right) = - \tfrac{3}{2} f^{IJK} F_{\nu\rho}{}^J \left(\frac{\delta I_1}{\delta C_{\mu\nu\rho}{}^I} \right) \doteq 0 \ , \qquad (3.10a)$$

$$0 \overset{?}{=} D_\rho \left(\frac{\delta I_1}{\delta C_{\mu\nu\rho}{}^I} \right) = - \tfrac{1}{3} m \left(\frac{\delta I_1}{\delta B_{\mu\nu}{}^I} \right) \doteq 0 \ . \qquad (3.10b)$$

These results are consistent with the δ_T, δ_U and δ_V-invariances of our action I_1, since

$$\begin{aligned} 0 = \delta_T I_1 = & - \alpha^I D_\mu \left(\frac{\delta I_1}{\delta A_\mu{}^I} \right) - m \alpha^I \Bigg[+ f^{IJK} \left\{ \overline{\lambda}^J \left(\frac{\delta I_1}{\delta \overline{\lambda}^K} \right) \right\} \\ & + f^{IJK} \left\{ \overline{\chi}^J \left(\frac{\delta I_1}{\delta \overline{\chi}^K} \right) \right\} - (T^I)^{jk} \left\{ \overline{\psi}^j \left(\frac{\delta I_1}{\delta \overline{\psi}^k} \right) \right\} \\ & + f^{IJK} B_{\mu\nu}{}^J \left(\frac{\delta I_1}{\delta B_{\mu\nu}{}^K} \right) + f^{IJK} C_{\mu\nu\rho}{}^J \left(\frac{\delta I_1}{\delta C_{\mu\nu\rho}{}^K} \right) \\ & + \frac{\delta I_1}{\{ (\delta e^\varphi) e^{-\varphi} \}^I} - (T^I)^{jk} A^j \left(\frac{\delta I_1}{\delta A^k} \right) \\ & - (T^I)^{jk} B^j \left(\frac{\delta I_1}{\delta B^k} \right) \Bigg] \ , \qquad (3.11a)\end{aligned}$$

$$0 = \delta_U I_1 = + 2 \beta_\mu{}^I \left[D_\nu \left(\frac{\delta I_1}{\delta B_{\mu\nu}{}^I} \right) + \tfrac{3}{2} f^{IJK} F_{\nu\rho}{}^J \left(\frac{\delta I_1}{\delta C_{\mu\nu\rho}{}^I} \right) \right] , \qquad (3.11b)$$

$$0 = \delta_V I_1 = - 3 \gamma_{\mu\nu}{}^I \left[D_\rho \left(\frac{\delta I_1}{\delta C_{\mu\nu\rho}{}^I} \right) + \tfrac{1}{3} m \left(\frac{\delta I_1}{\delta B_{\mu\nu}{}^I} \right) \right] \ . \qquad (3.11c)$$

As an important investigation, we solve the $A_\mu{}^I$-field equation (3.8d) for $A_\mu{}^I$ itself, and see its δ_T-transformation, as follows. First, from (3.8d), we get

$$\begin{aligned} A_\mu{}^I \doteq (M^{-1})^{IJ} \Big[& - m^{-1} \left\{ (\partial_\mu e^\varphi) e^{-\varphi} \right\}^J - \tfrac{1}{2} m^{-1} f^{JKL} (\overline{\chi}^K \gamma_\mu \chi^L) \\ & + \tfrac{1}{2} m^{-1} (T^J)^{kl} (\overline{\psi}^k \gamma_\mu \psi^\ell) \\ & + m^{-1} (A T^J \partial_\mu A) + m^{-1} (B T^J \partial_\mu B) \\ & + m^{-2} f^{JKL} \widetilde{G}_\mu{}^K (A T^L B) \Big] , \qquad (3.12)\end{aligned}$$

where $(A T^J \partial_\mu A) \equiv A^k (T^J)^{k\ell} \partial_\mu A^\ell$, $(A T^L B) \equiv A^j (T^L)^{jk} B^k$, *etc.*, and

$$M^{IJ} \equiv \delta^{IJ} - (A T^I T^J A) - (B T^I T^J B) \equiv \delta^{IJ} - N^{IJ} = M^{JI} \ . \qquad (3.13)$$

Second, by the useful lemmas, such as

$$\begin{aligned} & \delta_T N^{IJ} = - 2 f^{(I|KL} \alpha^K N^{L|J)} , \\ & \delta_T (M^{-1})^{IJ} = - 2 f^{(I|KL} \alpha^K (M^{-1})^{L|J)} \ , \\ & \delta_T \left[(A T^I \partial_\mu A) + (B T^I \partial_\mu B) \right] \\ & \quad = - m f^{IJK} \alpha^J \left[(A T^K \partial_\mu A) + (B T^K \partial_\mu B) \right] \\ & \qquad - m N^{IJ} \partial_\mu \alpha^J , \qquad (3.14)\end{aligned}$$

we can confirm the desirable transformation rule for (3.12):

$$\delta_T A_\mu{}^I \doteq \partial_\mu \alpha^I + m f^{IJK} A_\mu{}^J \alpha^K = D_\mu \alpha^I \ . \qquad (3.15)$$

The crucial point is that all gradient-terms with $\partial_\mu \alpha^I$ in δ_T [RHS of (3.12)] combine themselves to form a factor M^{IJ} to cancel $(M^{-1})^{IJ}$, yielding the unit strength in front of the gradient $\partial_\mu \alpha^I$.

The remaining terms form the homogeneous term $mf^{IJK}A_\mu{}^J\alpha^K$ again with the unit strength, with exactly the same expression of $A_\mu{}^I$ in the RHS of (3.12) itself.

As careful readers may have noticed, the $m(\bar\lambda\chi)$-term in our lagrangian (3.1) represents the crucial importance of the compensator mechanism. This $m(\bar\lambda\chi)$-term is a supersymmetric partner term for the minimal coupling in the kinetic term $-(1/2)(P_\mu{}^I)^2$. Because of this $m(\bar\lambda\chi)$-term, the λ-field equation (3.8a) has the first $m\chi$-term, that prohibits the vanishing of the remaining two terms in (3.8a). To be more specific, if the $m(\bar\lambda\chi)$-term *were* absent in the lagrangian, the λ-field equation *would* yield the constraint

$$m(T^I)^{jk}\Big[\psi^jA^k+i(\gamma_5\psi^j)B^k\Big]\doteq 0\ , \tag{3.16}$$

which *would* reduce the freedom of the A^i and/or B^j-fields, because (3.16) *would* be proportional to the supersymmetric variation

$$\delta_Q\Big[(T^I)^{jk}A^jB^k\Big]=(T^I)^{jk}\Big[(\bar\epsilon\gamma_5\psi^j)A^k+i(\bar\epsilon\psi^j)B^k\Big]\ . \tag{3.17}$$

Due to supersymmetry, this *would* imply the vanishing of $(T^I)^{jk}A^jB^k$ itself, which *would* reduce the freedom of A^i and B^i.

4. Gauged chiral-symmetry with composite gauge field

Our formulation has other examples. A good and simple application is the gauging of chiral-symmetry with $N=1$ supersymmetry in 4D [5]. This chiral-symmetry resembles the conventional R-symmetry [11,13], but is different. This is because our chiral-symmetry *does* commute with supersymmetry transformation δ_Q. The original *global* version of chiral symmetry is given with the parameter η in section 5 of the paper by Wess and Zumino [11].

To this end, our field content has three multiplets: (i) Abelian VM $(A_\mu,\lambda,C_{\mu\nu\rho})$, (ii) CTM $(B_{\mu\nu},\chi,\varphi)$, and (iii) CM (A,B,ψ,F,G). We consider the *local* Abelian chiral-symmetry for the CM with the transformation rule

$$\begin{aligned}&\delta_T(A,\,B,\,\psi,\,F,\,G)=m\alpha(-B,\,A,\,-i\gamma_5\psi,\,G,\,-F)\ ,\\&\delta_TA_\mu=+\partial_\mu\alpha\ ,\end{aligned}\tag{4.1}$$

with the infinitesimal real local parameter α. Accordingly, we define the T-covariant derivatives on the CM as

$$D_\mu A\equiv\partial_\mu A+mA_\mu B\ ,\qquad D_\mu B\equiv\partial_\mu B-mA_\mu A\ ,$$
$$D_\mu\psi\equiv\partial_\mu\psi+imA_\mu\gamma_5\psi\ , \tag{4.2a}$$
$$D_\mu F\equiv\partial_\mu F-mA_\mu G\ ,\qquad D_\mu G\equiv\partial_\mu G+mA_\mu F\ , \tag{4.2b}$$

transforming in a desirable fashion:

$$\delta_T(D_\mu A)=-m\alpha(D_\mu B)\,,\qquad \delta_T(D_\mu B)=-m\alpha(D_\mu A)\,,$$
$$\delta_T(D_\mu\psi)=+im\alpha\gamma_5(D_\mu\psi)\ , \tag{4.3a}$$
$$\delta_T(D_\mu F)=+m\alpha(D_\mu G)\,,\qquad \delta_T(D_\mu B)=+m\alpha(D_\mu A)\ . \tag{4.3b}$$

With these preliminaries, we now consider our total action $I_2\equiv m^2\int d^4\mathcal{L}_2$, where

$$\begin{aligned}\mathcal{L}_2=&-\tfrac{1}{12}(G_{\mu\nu\rho})^2+\tfrac12(\bar\chi\not{\partial}\chi)-\tfrac12P_\mu^2+m(\bar\lambda\chi)\\&-\tfrac12(D_\mu A)^2-\tfrac12(D_\mu B)^2+\tfrac12(\bar\psi\not{D}\psi)+\tfrac12F^2+\tfrac12G^2\\&-m(\bar\lambda\psi)A-im(\bar\lambda\gamma_5\psi)B+\tfrac12m\widetilde H(A^2+B^2)\ .\end{aligned}\tag{4.4}$$

As in the last section, we put *no* kinetic terms for the VM, since we are formulating a *composite* VM. The field strengths are essentially the same as in section 3, except that *non*-Abelian terms are absent:

$$F_{\mu\nu}\equiv+2\partial_{[\mu}A_{\nu]}{}^I\ ,\qquad G_{\mu\nu\rho}\equiv+3\partial_{[\mu}B_{\nu\rho]}+mC_{\mu\nu\rho}\ , \tag{4.5a}$$
$$H_{\mu\nu\rho\sigma}\equiv+4\partial_{[\mu}C_{\nu\rho\sigma]}\ ,\qquad \widetilde H\equiv\tfrac{1}{4!}\epsilon^{[4]}H_{[4]}\ ,$$
$$P_\mu\equiv+D_\mu\varphi\equiv\partial_\mu\varphi+mA_\mu\ . \tag{4.5b}$$

The $N=1$ supersymmetry transformation rule is

$$\delta_QA_\mu=+i(\bar\epsilon\gamma_5\gamma_\mu\lambda)\ ,$$
$$\delta_Q\lambda=\tfrac{i}{2}(\gamma_5\gamma^{\mu\nu}\epsilon)F_{\mu\nu}+\epsilon\widetilde H\ , \tag{4.6a}$$
$$\delta_QC_{\mu\nu\rho}=+i(\bar\epsilon\gamma_5\gamma_{\mu\nu\rho}\lambda)\ , \tag{4.6b}$$
$$\delta_QB_{\mu\nu}=+i(\bar\epsilon\gamma_5\gamma_{\mu\nu}\chi)\ ,\qquad \delta_Q\varphi=+i(\bar\epsilon\gamma_5\chi)\ , \tag{4.6c}$$
$$\delta_Q\chi=-\tfrac{i}{6}(\gamma_5\gamma^{[3]}\epsilon)G_{[3]}+i(\gamma_5\gamma^\mu\epsilon)P_\upsilon\ , \tag{4.6d}$$
$$\delta_QA=+(\bar\epsilon\psi)\ ,\qquad \delta_QB=+i(\bar\epsilon\gamma_5\psi)\ , \tag{4.6e}$$
$$\delta_Q\psi=-(\gamma^\mu\epsilon)D_\mu A+i(\gamma_5\gamma^\mu\epsilon)D_\mu B-\epsilon F-i(\gamma_5\epsilon)G\ , \tag{4.6f}$$
$$\delta_QF=+(\bar\epsilon\not{D}\psi)-m(\bar\epsilon\lambda)A-im(\bar\epsilon\gamma_5\lambda)B\ , \tag{4.6g}$$
$$\delta_QG=+i(\bar\epsilon\gamma_5\not{D}\psi)-im(\bar\epsilon\gamma_5\lambda)A+m(\bar\epsilon\lambda)B\ . \tag{4.6h}$$

Relevantly, the δ_U and δ_V-transformations on our fields are

$$\begin{aligned}&\delta_UB_{\mu\nu}=2\partial_{[\mu}\beta_{\nu]}\ ,\qquad \delta_VB_{\mu\nu}=-m\gamma_{\mu\nu}\ ,\\&\delta_VC_{\mu\nu\rho}=+3\partial_{[\mu}\gamma_{\nu\rho]}\ .\end{aligned}\tag{4.7}$$

The transformations of other fields, such as $\delta_UC_{\mu\nu\rho}$ are zero. All of our field strengths are invariant under (4.7): $\delta_U(F_{\mu\nu},G_{\mu\nu\rho},H_{\mu\nu\rho\sigma},P_\mu)=\delta_V(F_{\mu\nu},G_{\mu\nu\rho},H_{\mu\nu\rho\sigma},P_\mu)=(0,0,0,0)$.

The field equations of our system are

$$\begin{aligned}\frac{\delta I_2}{\delta A_\mu}&=-mP^\mu-mBD^\mu A+mAD^\mu B-\tfrac{i}{2}m(\bar\psi\gamma_5\gamma^\mu\psi)\\&\doteq0\ ,\end{aligned}\tag{4.8a}$$
$$\frac{\delta I_2}{\delta\bar\lambda}=+m\chi-im(\gamma_5\psi)B-m\psi A\doteq0\ , \tag{4.8b}$$
$$\frac{\delta I_2}{\delta C_{\mu\nu\rho}}=-\tfrac16m\Big[G^{\mu\nu\rho}-\tfrac12\epsilon^{\mu\nu\rho\sigma}\partial_\sigma(A^2+B^2)\Big]\doteq0\ , \tag{4.8c}$$
$$\frac{\delta I_2}{\delta B_{\mu\nu}}=+\tfrac12\partial_\rho G^{\mu\nu\rho}\doteq0\ ,$$
$$\frac{\delta I_2}{\delta\varphi}=+\partial_\mu P^\mu\doteq0\ ,\qquad \frac{\delta I_2}{\delta\bar\chi}=+\not{\partial}\chi+m\lambda\doteq0\ , \tag{4.8d}$$
$$\frac{\delta I_2}{\delta A}=+D_\mu^2A-m(\bar\lambda\psi)+m\widetilde H A\doteq0\ , \tag{4.8e}$$
$$\frac{\delta I_2}{\delta B}=+D_\mu^2B-im(\bar\lambda\gamma_5\psi)+m\widetilde H B\doteq0\ , \tag{4.8f}$$
$$\frac{\delta I_2}{\delta\bar\psi}=+\not{D}\psi-m\lambda A-im(\gamma_5\lambda)B\doteq0\ , \tag{4.8g}$$
$$\frac{\delta I_2}{\delta F}=+F\doteq0\ ,\qquad \frac{\delta I_2}{\delta G}=+G\doteq0\ . \tag{4.8h}$$

As in section 3, the consistencies of A_μ, $B_{\mu\nu}$ and $C_{\mu\nu\rho}$-field equations are associated with the δ_T, δ_U and δ_V-invariances of our action I_2:

$$\begin{aligned}&\partial_\mu\left(\frac{\delta I_2}{\delta A_\mu}\right)+mB\left(\frac{\delta I_2}{\delta A}\right)-mA\left(\frac{\delta I_2}{\delta B}\right)\\&\quad+im\bar\psi\gamma_5\left(\frac{\delta I_2}{\delta\bar\psi}\right)-mG\left(\frac{\delta I_2}{\delta F}\right)+mF\left(\frac{\delta I_2}{\delta G}\right)\equiv0\ ,\end{aligned}\tag{4.9a}$$
$$\partial_\nu\left(\frac{\delta I_2}{\delta B_{\mu\nu}}\right)\equiv0\ ,$$
$$\partial_\rho\left(\frac{\delta I_2}{\delta C_{\mu\nu\rho}}\right)+\tfrac13m\left(\frac{\delta I_2}{\delta B_{\mu\nu}}\right)\equiv0\ . \tag{4.9b}$$

It is not difficult to confirm (4.9) explicitly for our field equations in (4.7).

We can solve the A_μ-field equation (4.8a) for A_μ itself

$$A_\mu \doteq -m^{-1}(1+A^2+B^2)^{-1} \times \left[\partial_\mu \varphi + (B\partial_\mu A - A\partial_\mu B) - \frac{i}{2}(\overline{\psi}\gamma_5\gamma_\mu\psi) \right] \ . \qquad (4.10)$$

We can also confirm the desirable δ_T-transformation of this composite field:

$$\delta_T A_\mu \doteq \partial_\mu \alpha \ , \qquad (4.11)$$

by the use of $\delta_T(\partial_\mu\varphi) = -m\,\partial_\mu\alpha$, *etc.*

The importance of the compensator φ is also clear in (4.10). Because if φ *were* absent, the first unity-term in $(1+A^2+B^2)$ *would* disappear, and so *would* the $\partial_\mu\varphi$ in (4.10), and eventually A_μ *would* become singular in the limits $A \to 0$ and $B \to 0$. This *would* be a setback, because the perturbation around the zero-v.e.v. *would not* make sense.

Note also that our field strength of A_μ does *not* vanish, as seen perturbatively:

$$\begin{aligned}(4.10) \Longrightarrow A_\mu &\doteq -m^{-1}\left[\partial_\mu\varphi + B\partial_\mu A - A\partial_\mu B - \frac{i}{2}(\overline{\psi}\gamma_5\gamma_\mu\psi)\right] \\ &\quad + \mathcal{O}(\Phi^3) \\ \Longrightarrow F_{\mu\nu} &\doteq +4m^{-1}(\partial_{[\mu}A)(\partial_{\nu]}B) + im^{-1}(\overline{\psi}\gamma_5\gamma_{[\mu}\partial_{\nu]}\psi) \\ &\quad + \mathcal{O}(\Phi^3) \neq 0 \ . \end{aligned} \qquad (4.12)$$

Thus our A_μ in (4.10) can *not* be gauged away, and our *local* chiral-symmetry is solid symmetry, that does *not* disappear even for different gauge-frames.

5. Concluding remarks

In this work, we have studied supersymmetric composite gauge theories supplemented with a compensator mechanism. We have combined the three originally un-related formulations:

(i) Composite gauge field formulation.
(ii) Non-Abelian compensator formulation.
(iii) $N=1$ supersymmetrization.

This combination is also a natural extension of our recent work [3] on supersymmetric compensator mechanism. The work in [3] itself is based on the recent developments on tensor-hierarchy for non-Abelian tensors [6,7].

In the first model, we have given the composite gauge field formulation for $SO(\mathcal{N})$ with a composite scalar φ^I in the adjoint representation, consistent with $N=1$ supersymmetry. In our second model, we have applied a similar mechanism to chiral-symmetry with $N=1$ supersymmetry, and see the important role played by the compensator φ.

In the past, there was *no* strong motivation for dealing with *composite* gauge fields, because of the drawbacks, such as the singular limit prohibiting perturbation around 0-v.e.v.'s. This setback is now overcome by the compensator φ with supersymmetrization motivated by our recent work [12]. The total consistency of our system combining compensator fields and composite gauge multiplets is re-confirmed by $N=1$ supersymmetry.

As additional consistency, we have confirmed the δ_T-transformation of our composite gauge fields (3.12) and (4.10). These gauge fields have *non*-vanishing field strengths, so that our δ_T-gauge symmetry is *neither* pure-gauge, *nor* simply gauged away.

Some readers may wonder about the renormalizability of our theory. Such a question is legitimate, because our composite gauge fields (3.12) and (4.10) are infinite series. This is exemplified by the scalar field φ that enters as an exponential function. We argue that our system is renormalizable, as follows.

Before solving the ${A_\mu}^I$-field equation for ${A_\mu}^I$ itself, all interactions maintain the usual quadratic structure at the lagrangian level, as (2.5) for a non-supersymmetric case, or (3.12) and (4.10) for supersymmetric case show. The gauge-coupling constant m in section 3 has the dimension of $(\text{mass})^1$ in our convention, corresponding to the $(\text{mass})^0$ in the conventional field theory. Even though there are exponential couplings for the compensator φ^I, they are all related to non-Abelian interactions, but *not* to the composite feature. One easy way is to switch to an Abelian group, where all exponential factors disappear, but the composite mechanism still remains valid. From these viewpoints we regard our system as *renormalizable* in the conventional sense.

As for possible chiral anomaly for our 2nd model, it actually exists similar to triangular diagrams with three $\gamma_5\gamma_\mu$-factors for axial-vector gauge fields [14]. However, we can easily cancel it by doubling the CM, one with the coupling constant $+m$, and another with $-m$. This is because there is *no* mixed-vertex between these two CMs, so that the triangular anomaly diagrams exist for each CM separately with the strengths $+m^3$ and $-m^3$ canceling each other. Eventually, the quantum-level Ward-identity corresponding to (4.9a) is valid for the total system with a VM, a CTM, and two CMs.

There are two important aspects of our formulations: First, we now have the working examples of the aforementioned combination of composite gauge fields with non-Abelian compensators consistent with supersymmetry. Second, we have now explicit systems to formulate *local* gauge symmetry, where gauge fields are composite-fields with *no* physical degrees of freedom. These systems have desirable perturbations around zero-field limits. Such systems have great advantage, when the degrees of freedom are restricted by supersymmetry.

The combined system of composite gauge fields and scalar compensators may well provide potentially broad applications to both supersymmetry and supergravity in dimensions four and higher, superstring, M-theory, and other theories of extended objects [15].

Acknowledgement

We are grateful to J.H. Schwarz for important discussions.

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